arXiv:0912.3564v1 [math.GN] 18 Dec 2009

$\mathcal{SPM}$ BULLETIN

ISSUE NUMBER 29: December 2009 CE

Contents

1. Editor's note

The *Problem of the Issue* posed in Issue 11 (see at the end of the present Issue) was solved in Announcement 2.4 below.

The Borel version of the problem from Issue 23 (see below) was posed by Miller in his plenary lecture at the *IIIrd Workshop on Coverings, Selections, and Games in Topology.* This latter problem was also solved recently, by my student Tal Orenshtein and I. We hope to have a paper describing this solution ready during 2010.

Several additional problems from the *Open Problems in Topology* book chapter on SPM are being solved in several works in progress by several authors. This makes the civilian year 2009 very satisfactory in this respect.

Have a peaceful and fruitful 2010.

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2. Research announcements in "core" SPM

Note. The division between the present section and the next one is somewhat artificial, but perhaps still useful.

2.1. **Measurable cardinals and the cardinality of Lindelöf spaces.** If it is consistent that there is a measurable cardinal, then it is consistent that all points G_δ Rothberger spaces have "small" cardinality.

`http://arxiv.org/abs/0909.3663`

Marion Schepers

2.2. **Topological games and covering dimension.** We consider a natural way of extending the Lebesgue covering dimension to various classes of infinite dimensional separable metric spaces.

http://arxiv.org/abs/0909.5004

Liljana Babinkostova

2.3. **Menger's and Hurewicz's Problems: Solutions from "The Book" and refinements.** We provide simplified solutions of Menger's and Hurewicz's problems and conjectures, concerning generalizations of σ-compactness. The reader who is new to this field will find a self-contained treatment in Sections 1, 2, and 5.

Sections 3 and 4 contain new results, based on the mentioned simplified solutions. The main new result is that there is a set of reals X of cardinality $\mathfrak{b}$, which has the following property:

> Given point-cofinite covers $\mathcal{U}_1, \mathcal{U}_2, \dots$ of X, there are for each n sets $U_n, V_n \in \mathcal{U}_n$, such that each member of X is contained in all but finitely many of the sets $U_1 \cup V_1, U_2 \cup V_2, \dots$

This property is strictly stronger than Hurewicz's covering property, and by a result of Miller and the present author, one cannot prove the same result if we are only allowed to pick one set from each $\mathcal{U}_n$.

http://arxiv.org/abs/0909.5645

Boaz Tsaban

2.4. **Point-cofinite covers in the Laver model.** Let $\mathsf{S}_1(\Gamma, \Gamma)$ be the statement: For each sequence of point-cofinite open covers (i.e., γ-covers), one can pick one element from each cover and obtain a point-cofinite cover. $\mathfrak{b}$ is the minimal cardinality of a set of reals not satisfying $\mathsf{S}_1(\Gamma, \Gamma)$. We prove the following assertions:

(1) If there is an unbounded tower, then there are sets of reals of cardinality $\mathfrak{b}$, satisfying $\mathsf{S}_1(\Gamma, \Gamma)$.
(2) It is consistent that all sets of reals satisfying $\mathsf{S}_1(\Gamma, \Gamma)$ have cardinality smaller than $\mathfrak{b}$.

These results can also be formulated as dealing with Arhangel'skiĭ's property α_2 for spaces of continuous real-valued functions.

The main technical result is that in Laver's model, each set of reals of cardinality $\mathfrak{b}$ has an unbounded Borel image in the Baire space $\mathbb{N}^{\mathbb{N}}$.

http://arxiv.org/abs/0910.4063

Arnold W. Miller and Boaz Tsaban

2.5. **Projective versions of selection principles.** All spaces are assumed to be Tychonoff. A space X is called projectively P (where P is a topological property) if every continuous second countable image of X is P. Characterizations of projectively Menger spaces X in terms of continuous mappings $f : X \to \mathbb{R}^\omega$, of Menger base property with respect to separable pseudometrics and a selection principle restricted

to countable covers by cozero sets are given. If all finite powers of X are projectively Menger, then all countable subspaces of $C_p(X)$ have countable fan tightness. The class of projectively Menger spaces contains all Menger spaces as well as all σ-pseudocompact spaces, and all spaces of cardinality less than $\mathfrak{d}$. Projective versions of Hurewicz, Rothberger and other selection principles satisfy properties similar to the properties of projectively Menger spaces, as well as some specific properties. Thus, X is projectively Hurewicz if and only if $C_p(X)$ has the Monotonic Sequence Selection Property in the sense of Scheepers; βX is Rothberger iff X is pseudocompact and projectively Rothberger. Embeddability of the countable fan space V_ω into $C_p(X)$ or $C_p(X, 2)$ is characterized in terms of projective properties of X.

To appear in *Topology and its Applications*:

`http://dx.doi.org/10.1016/j.topol.2009.12.004`

Maddalena Bonanzinga, Filippo Cammaroto, Mikhail Matveev

3. Additional research announcements

3.1. **Some Ramsey theorems for finite n-colorable and n-chromatic graphs.** Given a fixed integer n, we prove Ramsey-type theorems for the classes of all finite ordered n-colorable graphs, finite n-colorable graphs, finite ordered n-chromatic graphs, and finite n-chromatic graphs.

`http://arxiv.org/abs/0908.0475`

L. Nguyen Van Thé

3.2. **Uniform convexity and the splitting problem for selections.** We continue to investigate cases when the Repovš-Semenov splitting problem for selections has an affirmative solution for continuous set-valued mappings. We consider the situation in infinite-dimensional uniformly convex Banach spaces. We use the notion of Polyak of uniform convexity and modulus of uniform convexity for arbitrary convex sets (not necessary balls). We study general geometric properties of uniformly convex sets. We also obtain an affirmative solution of the splitting problem for selections of certain set-valued mappings with uniformly convex images.

`http://arxiv.org/abs/0908.1216`

Maxim V. Balashov and Dušan Repovš

3.3. **The Solecki dichotomy for functions with analytic graphs.** A dichotomy discovered by Solecki says that a Baire class 1 function from a Souslin space into a Polish space either can be decomposed into countably many continuous functions, or else contains one particular function which cannot be so decomposed. In this paper we generalize this dichotomy to arbitrary functions with analytic graphs. We provide a "classical" proof, which uses only elementary combinatorics and topology.

`http://arxiv.org/abs/0908.1544`

Janusz Pawlikowski, Marcin Sabok

3.4. **A co-analytic maximal set of orthogonal measures.** We prove that if $V = L$ then there is a Π^1_1 maximal orthogonal (i.e. mutually singular) set of measures on Cantor space. This provides a natural counterpoint to the well-known Theorem of Preiss and Rataj that no analytic set of measures can be maximal orthogonal.

http://arxiv.org/abs/0908.1605

Vera Fischer, Asger Tornquist

3.5. **Model theory of operator algebras I: Stability.** Several authors have considered whether the ultrapower and the relative commutant of a C^*-algebra or II_1 factor depend on the choice of the ultrafilter. We show that the negative answer to each of these questions is equivalent to the Continuum Hypothesis, extending results of Ge-Hadwin and the first author.

http://arxiv.org/abs/0908.2790

Ilijas Farah, Bradd Hart, David Sherman

3.6. **Characterizing meager paratopological groups.** We prove that a Hausdorff paratopological group G is meager if and only if there are a nowhere dense subset A of G and a countable subset C in G such that $CA = G = AC$.

http://arxiv.org/abs/0908.2225

T. Banakh, I. Guran and A. Ravsky

3.7. **The topological structure of direct limits in the category of uniform spaces.** Let $(X_n)_n$ be a sequence of uniform spaces such that each space X_n is a closed subspace in X_{n+1}. We give an explicit description of the topology and uniformity of the direct limit $u - limX_n$ of the sequence (X_n) in the category of uniform spaces. This description implies that a function $f : u - limX_n \to Y$ to a uniform space Y is continuous if for every n the restriction $f|X_n$ is continuous and regular at the subset X_{n-1} in the sense that for any entourages $U \in \mathcal{U}_Y$ and $V \in \mathcal{U}_X$ there is an entourage $V \in \mathcal{U}_X$ such that for each point $x \in B(X_{n-1}, V)$ there is a point $x' \in X_{n-1}$ with $(x, x') \in V$ and $(f(x), f(x')) \in U$. Also we shall compare topologies of direct limits in various categories.

http://arxiv.org/abs/0908.2228

Taras Banakh

3.8. **Orthonormal bases of Hilbert spaces.** I prove that a Hilbert space has the property that each of its dense (not necessarily closed) subspaces contains an orthoormal basis if and only if it is separable.

http://arxiv.org/abs/0908.1942

Ilijas Farah

3.9. **A dichotomy for the Mackey Borel structure.** We prove that the equivalence of pure states of a separable C*-algebra is either smooth or it continuously reduces $[0, 1]^{\mathbb{N}}/\ell_2$ and it therefore cannot be classified by countable structures. The

latter was independently proved by Kerr–Li–Pichot by using different methods. We also give some remarks on a 1967 problem of Dixmier.

http://arxiv.org/abs/0908.1943

Ilijas Farah

3.10. **Minimal Size of Basic Families.** A family $\mathcal{B}$ of continuous real-valued functions on a space X is said to be *basic* if every $f \in C(X)$ can be represented $f = \sum_{i=1}^{n} g_i \circ \phi_i$ for some $\phi_i \in \mathcal{B}$ and $g_i \in C(\mathbb{R})$ $(i = 1, ..., n)$. Define basic$(X) = \min\{|\mathcal{B}| : \mathcal{B}$ is a basic family for $X\}$. If X is separable metrizable X then either X is locally compact and finite dimensional, and basic$(X) < \aleph_0$, or basic$(X) = \mathfrak{c}$. If K is compact and either $w(K)$ (the minimal size of a basis for K) has uncountable cofinality or K has a discrete subset D with $|D| = w(K)$ then either K is finite dimensional, and basic$(K) = \mathsf{cof}([w(K)]^{\aleph_0}, \subseteq)$, or basic$(K) = |C(K)| = w(K)^{\aleph_0}$.

http://arxiv.org/abs/0909.4563

Ziqin Feng and Paul Gartside

3.11. **On Hilbert's 13th Problem.** Every continuous function of two or more real variables can be written as the superposition of continuous functions of one real variable along with addition.

http://arxiv.org/abs/0909.4561

Ziqin Feng and Paul Gartside

3.12. **Metastability and the Furstenberg-Zimmer Tower II: Polynomial and Multidimensional Szemeredi's Theorem.** The Furstenberg-Zimmer structure theorem for $\mathbb{Z}^d$ actions says that every measure-preserving system can be decomposed into a tower of primitive extensions. Furstenberg and Katznelson used this analysis to prove the multidimensional Szemerédi's theorem, and Bergelson and Liebman further generalized to a polynomial Szemerédi's theorem. Beleznay and Foreman showed that, in general, this tower can have any countable height. Here we show that these proofs do not require the full height of this tower; we define a weaker combinatorial property which is sufficient for these proofs, and show that it always holds at fairly low levels in the transfinite construction (specifically, $\omega^{\omega^{\omega^{\omega}}}$).

http://arxiv.org/abs/0909.5668

Henry Towsner

3.13. **Cofinitary Groups and Other Almost Disjoint Families of Reals.** We study two different types of (maximal) almost disjoint families: very mad families and (maximal) cofinitary groups. For the very mad families we prove the basic existence results. We prove that MA implies there exist many pairwise orthogonal families, and that CH implies that for any very mad family there is one orthogonal to it. Finally we prove that the axiom of constructibility implies that there exists a coanalytic very mad family. Cofinitary groups have a natural action on the natural numbers. We prove that a maximal cofinitary group cannot have infinitely many orbits under this

action, but can have any combination of any finite number of finite orbits and any finite (but nonzero) number of infinite orbits.

We also prove that there exists a maximal cofinitary group into which each countable group embeds. This gives an example of a maximal cofinitary group that is not a free group. We start the investigation into which groups have cofinitary actions. The main result there is that it is consistent that $\bigoplus_{\alpha\in\aleph_1}\mathbb{Z}_2$ has a cofinitary action. Concerning the complexity of maximal cofinitary groups we prove that they cannot be K_σ, but that the axiom of constructibility implies that there exists a coanalytic maximal cofinitary group. We prove that the least cardinality $\mathfrak{a}_g$ of a maximal cofinitary group can consistently be less than the cofinality of the symmetric group. Finally we prove that $\mathfrak{a}_g$ can consistently be bigger than all cardinals in Cichoń's diagram.

http://arxiv.org/abs/0910.0279

Bart Kastermans

3.14. **Forcing, games and families of closed sets.** We propose a new, game-theoretic, approach to the idealized forcing, in terms of fusion games. This generalizes the classical approach to the Sacks and the Miller forcing. For definable ($\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$) σ-ideals we show that if a σ-ideal is generated by closed sets, then it is generated by closed sets in all forcing extensions. We also prove an infinite-dimensional version of the Solecki dichotomy for analytic sets. Among examples, we investigate the σ-ideal $\exists$ generated by closed null sets and σ-ideals connected with not piecewise continuous functions.

http://arxiv.org/abs/0910.2318

Marcin Sabok

3.15. **CH, a problem of Rolewicz and bidiscrete systems.** We give a construction under CH of a non-metrizable compact Hausdorff space K such that any uncountable semi-biorthogonal sequence in $C(K)$ must be of a very specific kind. The space K has many nice properties, such as being hereditarily separable, hereditarily Lindelöf and a 2-to-1 continuous preimage of a metric space, and all Radon measures on K are separable. However K is not a Rosenthal compactum.

We introduce the notion of bidiscrete systems in compact spaces and prove that every infinite compact Hausdorff space K must have a bidiscrete system of size $d(K)$, the density of K. This, in particular, implies that $C(K)$ has a biorthogonal system of size $d(K)$ (known for $d(K) = \aleph_1$) and the known result that K^2 has a discrete subspace of size $d(K)$.

http://arxiv.org/abs/0910.3091

Mirna Dzamonja, Istvan Juhasz

3.16. **A Note on Monotonically Metacompact Spaces.** We show that any metacompact Moore space is monotonically metacompact and use that result to characterize monotone metacompactness in certain generalized ordered (GO)spaces. We show, for example, that a generalized ordered space with a σ-closed-discrete dense subset is

metrizable if and only if it is monotonically (countably) metacompact, that a monotonically (countably) metacompact GO-space is hereditarily paracompact, and that a locally countably compact GO-space is metrizable if and only if it is monotonically (countably) metacompact. We give an example of a non-metrizable LOTS that is monotonically metacompact, thereby answering a question posed by S. G. Popvassilev. We also give consistent examples showing that if there is a Souslin line, then there is one Souslin line that is monotonically countable metacompact, and another Souslin line that is not monotonically countably metacompact.

`http://arxiv.org/abs/0910.4106`
Harold R. Bennett, Klaas Pieter Hart, David J. Lutzer

3.17. **Covering dimension and finite-to-one maps.** Hurewicz characterized the dimension of separable metrizable spaces by means of finite-to-one maps. We investigate whether this characterization also holds in the class of compact F-spaces of weight $\mathfrak{c}$. Our main result is that, assuming the Continuum Hypothesis, an n-dimensional compact F-space of weight $\mathfrak{c}$ is the continuous image of a zero-dimensional compact Hausdorff space by an at most 2^n-to-1 map.

`http://arxiv.org/abs/0910.4107`
Klaas Pieter Hart, Jan van Mill

3.18. **The Whyburn property in the class of P-spaces.** We investigate the Whyburn and weakly Whyburn property in the class of P-spaces, that is spaces where every G_δ set is open. We construct examples of non-weakly Whyburn P-spaces of size continuum, thus giving a negative answer under CH to a question of Pelant, Tkachenko, Tkachuk and Wilson. In addition, we show that the weak Kurepa Hypothesis (a set-theoretic assumption weaker than CH) implies the existence of a non-weakly Whyburn P-space of size $\aleph_2$. Finally, we consider the behavior of the above-mentioned properties under products; we show in particular that the product of a Lindelöf weakly Whyburn P-space and a Lindelöf Whyburn P-space is weakly Whyburn, and we give a consistent example of a non-Whyburn product of two Lindelöf Whyburn P-spaces.

`http://arxiv.org/abs/0911.0145`
Angelo Bella, Camillo Costantini and Santi Spadaro

3.19. **The Alaoglu theorem is equivalent to the Tychonoff theorem for compact Hausdorff spaces.** In this brief note we provide a simple approach to show that the Alaoglu theorem and the Tychonoff theorem for compact Hausdorff spaces are equivalent.

`http://arxiv.org/abs/0911.0332`
Stefano Rossi

3.20. **On splitting infinite-fold covers.** Let X be a set, κ be a cardinal number and let $\mathcal{A}$ be a family of subsets of X which covers each $x \in X$ at least κ times. What

assumptions can ensure that $\mathcal{A}$ can be decomposed into κ many disjoint subcovers? We examine this problem under various assumptions on the set X and on the cover $\mathcal{A}$: among other situations, we consider covers of topological spaces by closed sets, interval covers of linearly ordered sets and covers of $\mathbb{R}^n$ by polyhedra and by arbitrary convex sets. We focus on these problems mainly for infinite κ. Besides numerous positive and negative results, many questions turn out to be independent of the usual axioms of set theory.

http://arxiv.org/abs/0911.2774
Márton Elekes, Tamás Mátrai, Lajos Soukup

3.21. **Maximality of ideal-independent sets.** In this note we derive a property of maximal ideal-independent subsets of boolean algebras which has corollaries regarding the continuum cardinals p and $s_{mm}(P(\omega)/\text{fin})$.

http://arxiv.org/abs/0911.3861
Corey Thomas Bruns

3.22. **On Galvin's lemma and Ramsey spaces.** An abstract version of Galvin's lemma is proven, within the framework of the theory of Ramsey spaces. Some instances of it are explored.

http://arxiv.org/abs/0911.3833
Jose G. Mijares

3.23. **The topological structure of (homogeneous) spaces and groups with countable cs*-character.** In this paper we introduce and study three new cardinal topological invariants called the cs*-, cs-, and sb-characters. The class of topological spaces with countable cs*-character is closed under many topological operations and contains all $\aleph$-spaces and all spaces with point-countable cs*-network. Our principal result states that each non-metrizable sequential topological group with countable cs*-character has countable pseudo-character and contains an open k_ω-subgroup.

http://arxiv.org/abs/0911.4075
Taras Banakh and Lyubomyr Zdomskyy

3.24. **On topological groups containing a Fréchet-Urysohn fan.** Suppose G is a topological group containing a (closed) topological copy of the Fréchet-Urysohn fan. If G is a perfectly normal sequential space (a normal k-space) then every closed metrizable subset in G is locally compact. Applying this result to topological groups whose underlying topological space can be written as a direct limit of a sequence of closed metrizable subsets, we get that every such a group either is metrizable or is homeomorphic to the product of a k_ω-space and a discrete space.

http://arxiv.org/abs/0911.4081
Taras Banakh

3.25. **Hewitt-Marczewski-Pondiczery type theorem for abelian groups and Markov's potential density.** For an uncountable cardinal τ and a subset S of an abelian group G, the following conditions are equivalent:

(i) $|\{ns : s \in S\}| \geq \tau$ for all integers $n \geq 1$;
(ii) there exists a group homomorphism $\pi : G \to \mathbb{T}^{2^\tau}$ such that $\pi(S)$ is dense in $\mathbb{T}^{2^\tau}$.

Moreover, if $|G| \leq 2^{2^\tau}$, then the following item can be added to this list:

(iii) there exists an isomorphism $\pi : G \to G'$ between G and a subgroup G' of $\mathbb{T}^{2^\tau}$ such that $\pi(S)$ is dense in $\mathbb{T}^{2^\tau}$.

We prove that the following conditions are equivalent for an uncountable subset S of an abelian group G that is either (almost) torsion-free or divisible:

(a) S is $\mathcal{T}$-dense in G for some Hausdorff group topology $\mathcal{T}$ on G;
(b) S is $\mathcal{T}$-dense in some precompact Hausdorff group topology $\mathcal{T}$ on G;
(c) $|\{ns : s \in S\}| \geq \min\left\{\tau : |G| \leq 2^{2^\tau}\right\}$ for every integer $n \geq 1$.

This partially resolves a question of Markov going back to 1946.

`http://arxiv.org/abs/0912.0497`
Dikran Dikranjan, Dmitri Shakhmatov

3.26. **Product between ultrafilters and applications to the Connes' embedding problem.** In this paper we want to apply the notion of product between ultrafilters to answer several questions which arise around the Connes' embedding problem. For instance, we will give a simplification and generalization of a theorem by Radulescu; we will prove that ultraproduct of hyperlinear groups is still hyperlinear and consequently the von Neumann algebra of the free group with uncountable many generators is embeddable into R^ω. This follows also from a general construction that allows, starting from an hyperlinear group, to find a family of hyperlinear groups. We will introduce the notion of hyperlinear pair and we will use it to give some other characterizations of hyperlinearity. We will prove also that the cross product of a hyperlinear group via a profinite action is embeddable into R^ω.

`http://arxiv.org/abs/0911.4978`
V. Capraro, L. Paunescu

3.27. **A dichotomy for the number of ultrapowers.** We prove a strong dichotomy for the number of ultrapowers of a given countable model associated with nonprincipal ultrafilters on N. They are either all isomorphic, or else there are $2^{2^{\aleph_0}}$ many nonisomorphic ultrapowers. We prove the analogous result for metric structures, including C^*-algebras and II_1 factors, as well as their relative commutants and include several applications. We also show that the C^*-algebra $B(H)$ always has nonisomorphic relative commutants in its ultrapowers associated with nonprincipal ultrafilters on $\mathbb{N}$.

`http://arxiv.org/abs/0912.0406`

Ilijas Farah, Saharon Shelah

3.28. **Optimal Matrices of Partitions and an Application to Souslin Trees.** The basic result of this note is a statement about the existence of families of partitions of the set of natural numbers with some favourable properties, the n-optimal matrices of partitions. We use this to improve a decomposition result for strongly homogeneous Souslin trees. The latter is in turn applied to separate strong notions of rigidity of Souslin trees, thereby answering a considerable portion of a question of Fuchs and Hamkins.

`http://arxiv.org/abs/0912.0431`

Gido Scharfenberger-Fabian

3.29. **Locally minimal topological groups.** A Hausdorff topological group (G, τ) is called locally minimal if there exists a neighborhood U of 0 in τ such that U fails to be a neighborhood of zero in any Hausdorff group topology on G which is strictly coarser than τ. Examples of locally minimal groups are all subgroups of Banach-Lie groups, all locally compact groups and all minimal groups. Motivated by the fact that locally compact NSS groups are Lie groups, we study the connection between local minimality and the NSS property, establishing that under certain conditions, locally minimal NSS groups are metrizable.

A symmetric subset of an abelian group containing zero is said to be a GTG set if it generates a group topology in an analogous way as convex and symmetric subsets are unit balls for pseudonorms on a vector space. We consider topological groups which have a neighborhood basis at zero consisting of GTG sets. Examples of these locally GTG groups are: locally pseudo–convex spaces, groups uniformly free from small subgroups (UFSS groups) and locally compact abelian groups. The precise relation between these classes of groups is obtained: a topological abelian group is UFSS if and only if it is locally minimal, locally GTG and NSS. We develop a universal construction of GTG sets in arbitrary non-discrete metric abelian groups, that generates a strictly finer non-discrete UFSS topology and we characterize the metrizable abelian groups admitting a strictly finer non-discrete UFSS group topology. We also prove that a bounded abelian group G admits a non-discrete locally minimal and locally GTG group topology iff $|G| \geq \mathfrak{c}$.

`http://arxiv.org/abs/0912.2946`

Lydia Außenhofer, María Jesús Chasco, Dikran Dikranjan, Xabier Domínguez

4. Unsolved problems from earlier issues

Issue 1. Is $\binom{\Omega}{\Gamma} = \binom{\Omega}{\mathrm{T}}$?

Issue 2. Is $\mathsf{U}_{\mathrm{fin}}(\mathcal{O}, \Omega) = \mathsf{S}_{\mathrm{fin}}(\Gamma, \Omega)$? And if not, does $\mathsf{U}_{\mathrm{fin}}(\mathcal{O}, \Gamma)$ imply $\mathsf{S}_{\mathrm{fin}}(\Gamma, \Omega)$?

Issue 4. Does $\mathsf{S}_1(\Omega, \mathrm{T})$ imply $\mathsf{U}_{\mathrm{fin}}(\Gamma, \Gamma)$?

Issue 5. Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of $\mathfrak{p}^*$ in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying $\mathsf{S}_{\mathrm{fin}}(\mathcal{B}, \mathcal{B})$?

Issue 8. Does $X \notin \mathsf{NON}(\mathcal{M})$ and $Y \notin \mathsf{D}$ imply that $X \cup Y \notin \mathsf{COF}(\mathcal{M})$?

Issue 9 (CH). Is $\mathsf{Split}(\Lambda, \Lambda)$ preserved under finite unions?

Issue 10. Is $\mathsf{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of $\mathfrak{od}$ in that issue.)

Issue 11. Does $\mathsf{S}_1(\Gamma, \Gamma)$ always contain an element of cardinality $\mathfrak{b}$?

Problem 11 is solved. See Section 2.4 above.

Issue 12. Could there be a Baire metric space M of weight $\aleph_1$ and a partition $\mathcal{U}$ of M into $\aleph_1$ meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M?

Issue 14. Does there exist (in ZFC) a set of reals X of cardinality $\mathfrak{d}$ such that all finite powers of X have Menger's property $\mathsf{S}_{\mathrm{fin}}(\mathcal{O}, \mathcal{O})$?

Issue 15. Can a Borel non-σ-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there $X \subseteq \mathbb{R}$ of cardinality continuum, satisfying $\mathsf{S}_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?

Issue 17 (CH). Is there a totally imperfect X satisfying $\mathsf{U}_{\mathrm{fin}}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^{\mathbb{N}}$?

Issue 18 (CH). Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of $C_p(X)$ imply that X has Menger's property?

Issue 20. Does every hereditarily Hurewicz space satisfy $\mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$?

Issue 21 (CH). Is there a Rothberger-bounded $G \leq \mathbb{Z}^{\mathbb{N}}$ such that G^2 is not Menger-bounded?

Issue 22. Let $\mathcal{W}$ be the van der Waerden ideal. Are $\mathcal{W}$-ultrafilters closed under products?

Issue 23. Is the δ-property equivalent to the γ-property $\binom{\Omega}{\Gamma}$?

Previous issues. The previous issues of this bulletin are available online at
`http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22`

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